Enabling Opportunistic Search and Placement in Cache Networks

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Abstract

Content distribution networks have been extremely successful in today’s Internet. Despite their success, there are still a number of scalability and performance challenges that motivate clean slate solutions for content dissemination, such as content centric networking. In this paper, we address two of the fundamental problems faced by any content dissemination system: content search and content placement. We consider a multi-tiered, multi-domain hierarchical system wherein random walks are used to cope with the tradeoff between exploitation of known paths towards custodians versus opportunistic exploration of replicas in a given neighborhood. TTL-like mechanisms, referred to as reinforced counters, are used for content placement. We propose an analytical model to study the interplay between search and placement. The model yields closed form expressions for metrics of interest such as the average delay experienced by users and the load placed on custodians. Then, leveraging the model solution we pose a joint placement-search optimization problem. We show that previously proposed strategies for optimal placement, such as the square-root allocation, follow as special cases of ours, and that a bang-bang search policy is optimal if content allocation is given.
1. Introduction

Content distribution is in the vogue. Nowadays, virtually everybody can create, distribute and download content through the Internet. It is estimated that video distribution will alone account for up to 80% of global traffic by 2017 [1]. Despite the success of the current Internet infrastructure to support user demand, scalability challenges motivate clean slate approaches for content dissemination, such as information centric networking.

In information centric networks (ICNs), the focus is on content, rather than on hosts [2, 3]. Each content has an identification and is associated to at least one custodian. Once a request for a content is generated it flows towards a custodian through routers equipped with caches, referred to as cache-routers. (As such, ICNs are also referred to as cache networks. In this paper, we use both terms interchangeably.) A request that finds the content stored in a cache-router does not have to access the custodian. This alleviates the load at the custodians, reduces the delay to retrieve the content and the overall traffic in the network. To achieve performance gains with respect to existing architectures, in information centric networks cache-routers must efficiently and distributedly determine how to route content requests and where to place contents.

ICN architectures, such as NDN [2], are promising solutions for the future Internet. Still, it is unclear the scope at which the proposed solutions are feasible [4]. Incrementally deployable solutions are likely to prevail [5], and identifying the simplest foundational attributes of ICN architectures is essential while envisioning their Internet scale deployment.
The efficient management of distributed storage resources in the network coupled with the routing of requests for information retrieval are of fundamental importance [6, 7]. However, the interplay between search and placement is still not well understood, and there is a need to study search and placement problems under a holistic perspective. In fact, an adequate framework within which to assess the overall performance gains that ICNs can provide is still missing [6].

In this paper, we propose and study a simple cache-network architecture comprising of a logical hierarchy of cache-routers divided into tiers, where each tier is subdivided into one or more logical domains (Figure 1). In-between domains, requests are routed from users towards custodians which are assumed to be placed at the top of the hierarchy.

Figure 1: System diagram

To route content requests from users to custodians, a random lookup search takes place in the vicinity of the logically connected cache-routers (horizontal arrows in Figure 1). Cache-routers within a domain are assumed to form a logical *clique*. As such, a request that does not find the searched content in a cache-router is forwarded
to one of the remaining cache-routers in the same domain. The goal is to opportunistically explore the presence of content replicas in a given domain. If a copy is found in the domain within a reasonable time interval, the content is served. Otherwise, requests are routed from users towards custodians (vertical arrows in Figure 1). Custodians as well as the name resolution system (NRS) are supplied by third parties at the publishing area, and we focus our attention on the infrastructure from users to publishing areas.

By using random walks to opportunistically explore the presence of content replicas closer to users, we avoid content routing tables and tackle the scalability challenge posed in [8]. An alternative would be to adopt scoped-flooding [9]. However, scoped-flooding is more complex than random walks and requires some level of synchronization between caches. In addition, random walks have been shown to scale well in terms of overhead [10].

To efficiently and distributedly place content in the cache network, we consider a flexible content placement mechanism inspired by TTL caches. At each cache, a counter is associated to each content stored there, which we refer to as reinforced counter (RC). Whenever the RC surpasses a given threshold, the corresponding content is stored. The RC is decremented at a given established rate, until reaching zero, when the content is evicted.

Focusing on two of the simplest possible mechanisms for search and placement, namely random walks and TTL-like caches, our benefits are twofold. From a practitioners point of view, the proposed architecture is potentially deployable at the Internet scale [4]. From the performance evaluation perspective, our architecture is amenable to analytical treatment. Our quantitative analysis provides closed-form expressions for different metrics of interest, such as the average delay experienced by users.
Given such an architecture, we pose the following questions,

1. How long should the random-walk based search last at each domain so as to optimize the performance metrics of interest?

2. How should the reinforced counters be tuned so as to tradeoff content retrieval delay with server load at the custodian?

3. What parameters have the greatest impact on the performance metrics of the proposed cache network architecture?

To answer these questions, we introduce an analytical model that yields: a) the expected delay to find a content (average search time) and; b) the rate at which requests have to be satisfied by custodians. While the expected delay is directly related to users quality of experience, the rate of accesses towards the custodian is associated with publishing costs. The model yields simple closed-form expressions for the metrics of interest.

Using the model, we study different tradeoffs involved in the setting of the parameter values. In particular, we study the tradeoff between the time spent in opportunistic exploration around the vicinity of the user in order to find content and the custodian load.

In summary, our key contributions are the following:

**cache-network architecture:** we propose a simple multi-tiered cache network architecture based on random walks and TTL-like caches. Simplicity eases deployment and allows for analytical treatment, while capturing essential features of other cache-network architectures such as the tension between opportunistic exploration of replicas closer to users and exploitation of known paths towards custodians.
**Analytical model:** we introduce a simple analytical model of the proposed cache network architecture that can be helpful in the performance evaluation of ICNs. In particular, we consider the interplay between content placement and search. Using the model we show that we can achieve performance gains using a simple search strategy (random walks) and a logical hierarchical storage organization. Although our analysis is focused on the proposed architecture, we believe that the insights obtained are more broadly applicable to other architectures as well, such as scoped-flooding [9].

**Parameter tuning:** we formulate an optimization problem that leverages the closed-form expressions obtained with the proposed model to determine optimal search and placement parameters under storage constraints. We show that previously proposed strategies for optimal placement, such as the square-root allocation, follow as special cases of our solution, and that a bang-bang search policy is optimal if content allocation is given.

**Performance studies:** we investigate how different parameters impact system performance under different assumptions regarding the relative rate at which requests are issued and content is replaced in the cache-routers.

The remainder of this paper is organized as follows. After introducing background in Section 2, we describe the system studied in this paper in Section 3. An analytic model of this system is presented in Section 4. The joint placement and search optimization problem is posed and analyzed in Section 5 and numerical evaluations are presented in Section 6. Further discussions are presented in Section 7 and Section 8 concludes.
2. Background and Related Work

In this section we introduce the background used in this paper. We start by describing previous works on the modeling and analysis of cache networks. Then, in Section 2.2 we present previously proposed ICN architectures and in Section 2.3 we indicate some of the challenges they pose.

2.1. Modeling and Analysis of Cache Networks

The literature on the modeling and analysis of cache networks, accounting for TTL caches [11, 12], random replacement [13], complexity aspects [14] and optimal caching [15] is rapidly growing. There has been also recent efforts on determining unified frameworks to study and compare different replacement policies [16]. To the best of our knowledge, none of these previous works considered the joint problem of optimally determining search and placement strategies for TTL-like cache networks.

The performance of hierarchical caching systems has been analyzed in [12] and the advantages of pushing content upstream have been considered in [11]. The optimal placement problem has been considered, under heavy-tailed demands, in [17]. In this paper, in contrast, we address the search problem that is outside of the scope of previous studies.

Dehghan et al. [14] considered the complexity of jointly optimizing routing and placement, under a simple two-hop network. The authors show that the considered set of assumptions yields an NP-hard decision problem, and propose heuristics to solve special instances. In this work, in contrast, by considering an independence assumption among cache-routers, we were able to derive optimal search and placement policies.

Ioannidis and Yeh [15] proposed adaptive caching strategies for cache networks wherein nodes account for the costs to download content when making decisions
about which items to evict. The work is complementary to ours, as we consider
TTL-like caches and control the optimal cache parameters so as to minimize mean
download time and the load on the servers.

In this paper we assume that all contents have equal size. The impact of the
distribution of content sizes on performance has been studied in [18]. Adapting ideas
from [18] to the framework proposed in this paper, to account for the impact of
content sizes, is subject for future work.

Optimal content placement for unstructured peer-to-peer systems has been stud-
iied by Cohen and Shenker [19]. Whereas in unstructured peer-to-peer systems peers
are assumed to be fully connected, topology plays a key role in the study of cache
networks [20, 21, 18]. Accounting for topology poses a significant challenge. To
cope with this challenge, in this paper we consider hierarchically connected domains
wherein all nodes at each domain are logically fully connected. The models proposed
in this paper allow us to assess the impact of the hierarchical topology which charac-
terizes the inter-tier connections, as well as the distribution of the number of caches
per domain. They also allow us to appreciate the effects of load aggregation across
domains.

In this paper, we consider simple topologies to illustrate the applicability of our
model. Note that as we study virtual topologies for intra-domain connections, our
results are oblivious to the underlying physical topology inside each domain. The
effects of the real/physical topology were considered by [20, 21, 18, 9]. Studying
the impact of different virtual topologies beyond those considered in this paper, and
their interaction with physical topologies, is subject for future work.
2.2. ICN Architectures

This paper combines multi-tiered cache networks with random walks. Multi-tiered cache network designs have been considered in [22, 23], and random walks and related search strategies in [9, 10, 24], under fully connected topologies. We believe that combining the two contributes to the understanding of how ICN systems scale.

A survey comprising various architectures considered for ICN can be found in [6]. In what follows, we focus on five of the prominent architectures, namely DONA, PSIRP, Netinf, Multicache and NDN, which are most relevant to our work.

DONA [3] consists of a hierarchy of domains. Each domain includes a logical Resolution Handler (RH) that tracks the contents published in the domain and in the descendant domains. Therefore, the logical RH placed in the highest level of the hierarchy is aware of all the content published in the entire network. RHs provide a hierarchical name resolution service over the routing infra-structure. DONA supports caching through the RH infrastructure. When a RH aims at storing a content, it replaces the IP address of the requester by its own IP address. Then, the content will be delivered first to the RH before being forwarded to the end users, allowing the RH to cache the content within the domain.

PSIRP [25], Netinf [26] and Multicache [27] handle name resolution through a set of Request Nodes (RNs) organized according to a hierarchical Distributed Hash Table (DHT). Content is sent to the user through a set of forward nodes (FNs), under a separate network. FNs can advertise cached information to RNs to enhance the search efficiency and cache hit ratio. Nonetheless, as RNs cannot keep track of all replicas within the network, a key challenge consists of determining what is the relevant information to advertise.

The NDN [2] architecture handles name resolution using content routing tables. Users issue Interest messages to request a content. Messages are forwarded hop-by-
hop by Content Routers (CRs) until the content is found. Messages leave a trail of bread crumbs and the content follows the reverse path set by the trail. As content flows to requesters, the bread crumbs are removed. Published content is announced through routing protocols, with routing tables supporting name aggregation (names are hierarchical). To enhance the discovery of cached contents, Rosensweig et al. [28] allow bread crumbs not to be consumed on the fly when content traverses the network. This allows trails for previously downloaded contents to be preserved.

2.3. Challenges

Some of the main challenges faced by present ICN architectures are discussed in [8]. For Name Resolution Services (NRS) lookup proposals, such as Dona, NetInf and PSIRP, the challenge is to build a scalable resolution system which provides: (i) fast mapping of the name of the content to its locators; (ii) fast update of the location of a content since locations can change frequently; (iii) an efficient scheme to incorporate copies of a content in the cache routers.

For proposals based on content routing tables, such as NDN, the number of contents may be around $10^{15}$ to $10^{22}$. Routing table design becomes a challenge as its size is proportional to the number of contents in the system. Route announcements due to replica updates, and link failures, pose additional challenges.

To face the scalability challenge related to content routing tables, [9] proposes the use of flooding in each neighborhood, which simplifies design and reduces complexity. In this paper, in contrast, we propose the use of random walks. Random walks are as simple as flooding, and lead to reduced congestion [10, 29, 30] while still taking advantage of spacial and temporal locality [31].

For proposals relying on DHTs there exist many unsolved security vulnerabilities that are able to disrupt the pre-defined operation of DHT nodes [32] and need to be
overcome. Note that in a network composed of domains where providers care about administrative autonomy, the use of a global hash table becomes unfeasible [23].

3. Cache-network Architecture

In this section we describe the system architecture considered in this paper. We begin with a brief overview.

3.1. Tiers and Domains

The system consists of a set of cache-routers partitioned into several logical domains, which are organized into hierarchically arranged tiers (Figure 1). Each domain consists of a set of routers or cache-routers that are responsible for forwarding requests and caching copies of contents. In what follows, we assume that all routers are equipped with caches, and use interchangeably the terms router and cache-router.

Users generate requests at the lowest level of the hierarchy. These requests flow across domains, following the tier hierarchy towards the publishing areas, at the top of the hierarchy. Figure 1 displays routers forwarding requests towards a publishing area (green arrows). We consider $M$ logical hierarchical tiers. Tier 1 is the top level tier and tier $M$ is the bottom level constituted by routers that are “closest” to the users, i.e., which are the first to receive requests from users. The publishing area knows how to forward a request to a publisher in case the content is not found in any of the tiers. We adopt a strategy that allows opportunistic encounters between requests and replicas in a best-effort manner.

Each cache maintains a counter (one per content), referred to as a reinforced-counter, to establish thresholds to guide content placement at the caches. Copies of popular contents may be cached in the routers. Whenever a request arrives to a
domain, it generates a random walk to explore the domain, so as to allow opportunistic encounters with the desired content, taking advantage of the temporal and geographical correlations encountered by popular requests [31]. We rely on random walks in order to avoid the control overhead associated to routing table updates and the drawbacks of DHTs discussed in the previous section.

3.2. Random Walk Search

Random walks are one of the simplest search mechanisms with the flexibility to account for opportunistic encounters between user requests and replicas stored within the domains (purple arrows in Figure 1). Opportunistic encounters satisfy requests without the need for them to reach the publishing area. A request that reaches the publishing area indicates that the corresponding content was not found in any of the domains traversed by it.

When a request arrives to a domain, if the cache-router that receives the request does not have the content, it starts a random walk search. The random walk lasts for at most $T$ units of time, only traversing routers in the domain. A time-to-live (TTL) counter is set to limit the amount of search time for a content within a domain. If the content has not been found by the time the TTL counter expires, the router that holds the request transfers it to the next tier above it in the hierarchy.

When content is located in the network, two actions are performed: (a) the content is sent to the requester, following the reverse path of the request, i.e., through reverse path forwarding (blue arrows in Figure 1) and (b) the content is possibly stored in the caches of the routers that first received the request in each domain (those that initiated the random walk at a domain). Note that a cache-router may store contents that were found either in its own domain or in tiers above it. In the analysis that follows, cache occupancies are determined only by demand, and are not
affected by the mechanisms implemented in the downstream.

Action (b) is performed if the reinforced counters associated with the given content at the considered cache-routers reach a pre-determined threshold. The action implies that, in any given tier, at most one additional copy of a content is stored per miss. This is motivated by the importance of maintaining content diversity in the network. In addition, storing the content at multiple cache-routers per domain may increase content churn, is not scalable and might decrease performance [15, 10].

3.3. Reinforced Counter Based Placement

We consider a special class of content placement mechanisms, henceforth referred to as reinforced counters (RC), similar in spirit to TTL-caches [33].

Each published content in the network is identified by a unique hash key inf. Each cache-router has a set of RCs, one for each content being stored in the router. Reinforced counters are affected by exogenous requests and interdomain requests, but not by endogenous requests inside a given domain, that is, their values are not altered by the random walk search.

At any cache, the reinforced-counter associated to a given content is incremented by one at every exogenous or interdomain request to that content, and is decremented by one at every tick of a timer. The timer ticks at a rate of $\mu$ ticks per second.

Associated with each RC is a threshold $K$. Whenever a request for content inf reaches a router, either (i) an already pre-allocated counter for inf is incremented by one in this router or (ii) a new RC is allocated for inf and set to one. If the value of the RC surpasses $K$, the content is stored after inf is found.

RCs are decremented over time. Whenever the RC for inf is decremented from $K + 1$ to $K$ the content is evicted from the cache. The counter is deallocated when it reaches zero.
Note that, assuming independent Poisson request streams (this assumption will be justified in Section 4), the RC dynamics of different contents are uncoupled and the RC values are independent of each other. Cache storage constraints are taken into account in the model by limiting the average number of replicas in each cache, which corresponds to soft constraints. Since hard constraints on the cache occupancy must be enforced, the RC threshold should be set in such a way that the probability of a cache overflow is small [34]. By limiting the fraction of time that each content is cached, reinforced counters take advantage of statistical multiplexing of contents in the system.

3.4. Stateless and Stateful Searches

We consider two variants of random walk searches: stateless and stateful. Under stateless searches, requests do not carry any information about previously visited cache-routers. In other words, when a cache-router is visited, the only information that is known is the content of the cache currently being visited. In a stateful search requests either a) remember the cache-routers that have been visited or b) know ahead of time what routers to visit. We assume that in stateful searches the searcher never revisits cache-routers. The stateless and stateful searches are studied in Sections 4.2.1 and 4.2.2, respectively.

4. Analytical Model

In this section we present an analytical model to obtain performance metrics for the cache-network architecture described in the previous section, illustrated in Figure 1. The model takes into account the performance impact of content search through random walks and the cache management mechanism based on reinforced counters.
In particular, the model allows one to compute the probability of finding a content in a domain and the mean time to find it. Using the model, we show the benefit of a hierarchical structure and study the tradeoff between the storage requirements of the cache-routers and the load that reaches the publishing area.

When a request reaches a cache-router, the local cache is searched and if the content is locally stored it is immediately retrieved and sent to the user. If the content is not found, a random walk search starts in the domain. We assume that the random search takes $V$ time units per each cache-router visited where $V$ is an exponentially distributed random variable with rate $\gamma$.

Long search times can have an adverse effect on performance; hence, a timer is set when the random walk starts to limit the search time. The search can last for at most $T$ time units. The search ends when the timer expires or the content is found, whichever occurs first. As described in Section 3.2, if the timer expires the user request is sent to the next cache-router in the tier hierarchy, and the process restarts. Table 1 summarizes the notation used in the remainder of this paper.

4.1. Cache hit and insertion ratios

Consider a given tagged tier and cache-router in this tier. We assume that requests to content $c$ arrive to this cache-router according to a Poisson process with rate $\lambda_c$. $\lambda_c$ is also referred to as the content popularity. Recent work [35] using three months of data collected from the largest VoD provider in Brazil indicates that, during peak hours, the Poisson process is well suited to model the video request arrival process. Although, according to measurements, the Poisson process accurately models the traffic for a given content that arrives in the domains of the first tiers (tier $M$), the traffic for the remaining upper tier domains (in the case of multiple tier structures) is not Poisson. However, for model tractability, we assume that requests
Table 1: Table of notation. Note that subscripts are omitted when clear from context.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1/\gamma$</td>
<td>average delay per-hop, i.e., average time for the random walk to check for a content at a cache and possibly forward the request</td>
</tr>
<tr>
<td>$C(\Lambda_c)$</td>
<td>cost incurred by custodian (measured in delay experienced by users)</td>
</tr>
<tr>
<td>$C$</td>
<td>number of contents</td>
</tr>
<tr>
<td>$M$</td>
<td>number of tiers</td>
</tr>
<tr>
<td>$N$</td>
<td>number of caches in domain under consideration</td>
</tr>
<tr>
<td>$\lambda_{c,i}$</td>
<td>arrival rate of exogenous and interdomain requests for content $c$ at typical cache of domain $i$, $\lambda = \sum_{i=1}^{M} \sum_{c=1}^{C} \lambda_{c,i}$</td>
</tr>
<tr>
<td>$\Lambda_c$</td>
<td>exogenous arrival rate of requests for $c$ at the network (except otherwise noted, exogenous requests are issued at tier $M$)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_c$</td>
<td>number of replicas of content $c$ in tagged tier</td>
</tr>
<tr>
<td>$\pi_{c,i}$</td>
<td>probability that content $c$ is stored at typical cache at domain $i$</td>
</tr>
<tr>
<td>$\alpha_{c,i} = 1/\mu_{c,i}$</td>
<td>reinforced counter decrement rate for content $c$ at domain $i$</td>
</tr>
<tr>
<td>$T_{c,i}$</td>
<td>TTL for content $c$ at domain $i$, i.e., maximum time to perform a random search for content $c$ at domain $i$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Metric</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{c,i}(t)$</td>
<td>probability of not finding content $c$ at tier $i$ by time $t$</td>
</tr>
<tr>
<td>$D_{c,i}$</td>
<td>delay incurred for finding content $c$ at tier $i$</td>
</tr>
<tr>
<td>$D_c$</td>
<td>delay incurred for finding content $c$</td>
</tr>
<tr>
<td>$\hat{\Lambda}_c$</td>
<td>rate of requests for content $c$ at the publisher</td>
</tr>
</tbody>
</table>

for any content that flows from a domain to the next in the tier hierarchy is also Poisson. This assumption was motivated by the assumptions and discussions in [28] and the conclusions in [36] for cache networks with tree-structures. In addition, our own simulation studies reported in Appendix D provide evidence that this is indeed a reasonable assumption to make. In our numerical experiments, we rely on the Pois-
son assumption coupled with the Zipf distribution for popularities to characterize the workload.

We recall from Section 3.3 that the reinforced counter associated to a given content $c$ is incremented at every request for $c$ and decremented at constant rate $\mu_c$. We assume that the counter is decremented at exponentially distributed times with mean $1/\mu_c$. Associated with each counter and content is a threshold $K_c$ such that when the counter exceeds $K_c$, content $c$ must be stored into cache. Let $\pi_c$ denote the probability that the cache-router contains content $c$. Due to the assumption of Poisson arrivals and exponential decrement times, the dynamics of each reinforced counter is characterized by a birth-death process. Hence $\pi_c$, which is the probability that the reinforced counter has value greater than $K_c$, is given by

$$\pi_c = \left(\frac{\lambda_c}{\mu_c}\right)^{K_c+1}$$

If $K_c = 0$ we have $\pi_c = \lambda_c/\mu_c$, which we denote by $\rho_c$.

Let $\beta_c$ denote the miss rate for content $c$. Then,

$$\beta_c = \lambda_c(1 - \pi_c)$$

In Appendix A we consider an additional metric of interest, namely the cache insertion rate, which is the rate at which content is inserted into cache. Note that the cache insertion rate is lower than the cache miss rate, as not all misses lead to content insertions. We show that larger values of $K_c$ yield lower insertion rates, which translate into less overhead due to content churn. Despite the advantages of using $K_c > 0$, without loss of generality, and to facilitate the exposition, in the remainder of this paper we assume $K_c = 0$, except otherwise noted.
4.2. Publisher Hit Probability

We start by considering a single domain in a single tiered hierarchy, wherein \(N\) cache-routers are logically fully connected, i.e., any cache-router can exchange messages with any other router in the same domain. Our goal is to compute the probability \(R(t)\) that a random walk does not find the requested content by time \(t\), \(t > 0\). Note that \(R(T_c)\) equals the probability that the request is forwarded to the custodian.

We consider two slightly different models. As in the previous section, both models assume that requests for a content arrive according to a Poisson process. In what follows we describe the assumptions associated with each model, and comment on their applicability. In Sections 4.2.1 and 4.2.2 the analysis of stateless and stateful searches focuses on a tagged content \(c\).

4.2.1. Model 1: Stateless search

Recall that a stateless search is a search in which requests do not carry any information about previously visited cache-routers. We assume that searches are sufficiently fast so that the probability that content placement in a domain changes during the search is negligible. This assumption is reasonable if the expected time it takes for the random walker to check for the presence of content \(c\) in a cache and to transit from a cache-router to another, \(1/\gamma\), is very small compared to the mean time between: (a) two requests for \(c\), \(1/\lambda_c\), and; (b) decrements of the reinforced counter for \(c\), \(1/\mu_c\).

When an inter-domain request for a given content \(c\) arrives at a cache-router and a miss occurs, a random stateless search for \(c\) starts. After each visit to a cache-router, if the content is not found another cache-router is selected uniformly at random among the remaining \(N - 1\) cache-routers. Note that, because the search
is stateless, nodes can be revisited during the search.

In Section 3.3 we discussed the decoupling between RCs of different contents in a given cache. Next, we argue that RCs for different caches in a domain can also be treated independently. Recall that reinforced counters are not affected by endogenous requests inside a given domain, so we restrict ourselves to the impact of inter-domain requests when studying cache occupancies. Due to symmetry, we assume that the request rates from outside of a domain for a given content at different cache-routers in a domain are identical. Due to the Poisson assumption, a request for content c that arrives at a tagged cache-router sees the system in equilibrium (PASTA property). Therefore, arrivals will find the content of interest at a given cache with probability $\pi_c$, independent of the state of the neighboring caches in that domain.

Let $L_c$ be the random variable equal to the number of replicas of the content c in the domain, excluding the router being visited. We have:

$$P(L_c = l) = \binom{N - 1}{l} \pi_c^l (1 - \pi_c)^{N-1-l}. \quad (3)$$

Let $J_c$ denote the number of hops traversed by the stateless request by time $t$. Since the time between visits is assumed to be exponentially distributed,

$$R(t|J_c = j, L_c = l) = (1 - \pi_c)(1 - w_l)^j \quad (4)$$

where $w_l$ is the conditional probability that the random walker selects one router with content c from the remaining $N - 1$ routers in the domain when there are l replicas of the content in the domain given that the current router does not have the content. Then, $w_l = l/(N - 1)$. Note that $\pi_c$ depends on the placement policy and is defined partially by its parameter values.

**Proposition 4.1.** The probability $R_c(t|L_c = l)$ is given by

$$R_c(t|L_c = l) = (1 - \pi_c)e^{-\gamma w_l t} \quad (5)$$
Proof: From (4) we have:

\[ R_c(t|L_c = l) = (1 - \pi_c) \sum_{n=0}^{\infty} \frac{(\gamma t)^n}{n!} (1 - \omega l)^n e^{-\gamma t} \]

\[ = \frac{1 - \pi_c}{e^{\gamma t \omega l}} \sum_{n=0}^{\infty} \frac{(\gamma t(1 - \omega l))^n}{n!} e^{-\gamma t(1 - \omega l)} \]

\[ = (1 - \pi_c) e^{-\gamma t} \]  

(6)

□

Proposition 4.2 (Stateless search). The probability \( R_c(t) \) that a walker does not find a requested tagged content in a domain by time \( t \) is given by:

\[ R_c(t) = (e^{-\gamma t/(N-1)} \pi_c + (1 - \pi_c))^{(N-1)} (1 - \pi_c) \]  

(7)

Proof: Unconditioning (5) on \( L_c \), yields

\[ R_c(t) = \sum_{l=0}^{N-1} R(t|L_c = l) \binom{N-1}{l} \pi_c^l (1 - \pi_c)^{(N-1)-l} \]

\[ = (1 - \pi_c) \sum_{l=0}^{N-1} \frac{e^{-\gamma t l}}{l!} \binom{N-1}{l} \pi_c^l (1 - \pi_c)^{(N-1)-l} \]

\[ = (1 - \pi_c) \sum_{l=0}^{N-1} \left( \frac{N-1}{l} \right) (e^{-\gamma t/(N-1)} \pi_c)^l (1 - \pi_c)^{(N-1)-l} \]

\[ = (1 - \pi_c) \left( \pi_c e^{-\gamma t/(N-1)} + (1 - \pi_c) \right)^{(N-1)} \]  

(8)

□

According to (7) (large values of \( N \)), \( R_c(\infty) = (1 - \pi_c)^N \). As \( t \) increases, the probability that the walker does not find content \( c \) approaches the probability that all \( N \) caches within the domain do not hold the content.
4.2.2. Model 2: Stateful search

In this section, we consider stateful searches wherein requests remember the cache-routers that have been visited, i.e., after the search is initiated, the searcher chooses the next router to visit uniformly at random, from those that have not yet been visited before. Alternatively, requests know ahead of time what routers to visit. This latter approach is discussed in Appendix C.

Under a stateful search, the searcher never revisits cache-routers. This is possible because cache-routers are logically fully-connected. As in the stateless model, we assume that arrivals of inter-domain requests for content $c$ at cache-routers are characterized by Poisson processes. Therefore, the random searches for $c$ that are initiated at a tagged router $i$ are characterized by a Poisson process modulated by the RC of router $i$, whose dynamics is governed by a birth-death Markovian process. It is shown in [37] that the PASTA property holds for Poisson processes modulated by independent Markovian processes. Therefore, a search that starts at router $i$ and arrives at router $k \neq i$ sees the RC at $k$ in equilibrium, i.e., the request issued at router $i$ finds the desired content at cache $k$ with probability $\pi_c$. Conditioning on $J_c = j$ hops being traversed by time $t$, the probability that content $c$ is not found is given by

$$\tilde{R}_c(t|J_c = j) = (1 - \pi_c)^{j+1}$$

(9)

It remains to remove the conditioning on $J_c$.

We assume, as in the stateless model, that the search takes an exponentially distributed random delay at each hop, independent of the system state.

**Proposition 4.3 (Stateful search).** The probability $\tilde{R}(t)$ that a tagged content is not found by a stateful search by time $t$ is given by

$$\tilde{R}_c(t) = (1 - \pi_c)(e^{-\gamma \pi_c t} + g(N))$$

(10)
where
\[
g(N) = (1 - \pi_c)^{N-1} \sum_{n=N}^{\infty} \frac{(\gamma t)^n}{n!} e^{-\gamma t} (1 - (1 - \pi_c)^{n+1-N})
\] (11)

Proof: The proof is similar to that of Proposition 4.1. The time between cache visits is an exponential random variable with rate \( \gamma \). It follows from (9) that
\[
\tilde{R}_c(t) = \sum_{n=0}^{N-1} \tilde{R}_c(t|J = n) \frac{(\gamma t)^n}{n!} e^{-\gamma t} + \tilde{R}_c(t|J = N - 1) \sum_{n=N}^{\infty} \frac{(\gamma t)^n}{n!} e^{-\gamma t}
\] (12)
\[
= (1 - \pi_c) \left( \sum_{n=0}^{N-1} \frac{(\gamma t)^n}{n!} e^{-\gamma t} (1 - \pi_c)^n + (1 - \pi_c) \sum_{n=N}^{\infty} \frac{(\gamma t)^n}{n!} e^{-\gamma t} \right)
\]
\[
= (1 - \pi_c) \sum_{n=0}^{\infty} \frac{((1 - \pi_c)\gamma t)^n}{n!} e^{-\gamma (1-\pi_c)t} + g(N)
\]
\[
= (1 - \pi_c) (e^{-\gamma \pi_c t} + g(N))
\] (13)

For large values of \( N \), it follows from Proposition 4.3 that
\[
\tilde{R}_c(t) \approx (1 - \pi_c) e^{-\gamma \pi_c t}
\] (14)

The validity of the large \( N \) assumption can be checked by using the Normal distribution approximation for the Poisson distribution. For instance, the sum \( \sum_{n=N}^{\infty} \frac{(\gamma t)^n}{n!} e^{-\gamma t} \) that appears in the expression of \( g(N) \) is well approximated by the complementary cumulative distribution of the Normal distribution, \( 1 - \Phi \left( \frac{N - \gamma t}{\sqrt{\gamma t}} \right) \), for values of \( N > \gamma t + 4 \sqrt{\gamma t} \), where \( \Phi(x) \) is the cumulative distribution function of the standard Normal distribution.

According to (14), \( \tilde{R}_c(\infty) = 0 \). As the random walk progresses, contents are dynamically inserted and evicted from the caches and the walker eventually finds the desired content.
4.2.3. Multi-tier Networks

In the previous sections we considered a single tiered network. In what follows we extend these results to the multi-tier case. In Section 6 we discuss the potential performance benefits of a multi-tiered architecture.

We recall that in Section 4.1, we assumed that the requests at any domain in the tier hierarchy arrive according to a Poisson process. (See also Appendix D for further discussion on this assumption.) Refer to Figure 1 and let $M$ denote the number of tiers. Let $\hat{\Lambda}_c$ denote the publisher load accounting for the requests filtered at the $M$ tiers. Let $R_{c,i}(T_{c,i})$ denote the probability that a search that reaches domain $i$ fails to find content $c$ at that domain. The load for content $c$ that arrives at the publishing area is given by:

$$\hat{\Lambda}_c = \Lambda_c \prod_{i=1}^{M} R_{c,i}(T_{c,i})$$

where $\prod_{i=1}^{M} R_{c,i}(T_{c,i})$ is the probability that a request arrives at the publishing area and $\Lambda_c$ is the load generated by the users for content $c$ which are all placed at tier $M$. Note that replacing $R_{c,i}(T_{c,i})$ by $\tilde{R}_{c,i}(T_{c,i})$ corresponds to using the stateful model in place of the stateless one.

4.3. Average Delay

Let $D_{c,i}$ be the random variable denoting the delay experienced by requests for content $c$ at domain $i$, if content $c$ is found in domain $i$ by time $T_{c,i}$; $D_{c,i} = T_{c,i}$, if the content is not found by $T_{c,i}$. Let $\mathbb{I}(t)$ be the indicator function defined as follows: (a) for $t \leq T_{c,i}$ it is equal to 1 if the content that is being searched for is not found by $t$ and 0 otherwise; (b) for $t > T_{c,i}$, $\mathbb{I}(t) = 0$. Clearly, $P[\mathbb{I}(t) = 1] = R_{c,i}(t)$. 

23
Then (see [38]), making the dependence of $D_{c,i}$ on $T_{c,i}$ explicit:

$$E[D_{c,i}(T_{c,i})] = \int_0^{\infty} E[I(s) = 1] ds = \int_0^{T_{c,i}} P[I(s) = 1] ds$$

$$= \int_0^{T_{c,i}} R_{c,i}(s) ds. \tag{16}$$

Under the stateless model, $E[D_{c,i}(T_{c,i})]$ does not admit a simple closed form solution and must be obtained through numerical integration of (7). On the other hand, when the stateful model is employed, we obtain, after replacing (14) into (16),

$$E[D_{c,i}(T_{c,i})] = \int_0^{T_{c,i}} (1 - \pi_{c,i}) e^{-\gamma \pi_{c,i} t} dt \tag{17}$$

$$= (1 - \pi_{c,i}) \frac{1 - e^{-\gamma \pi_{c,i} T_{c,i}}}{\pi_{c,i} \gamma}. \tag{18}$$

Let $D_c$ denote the delay to find content $c$, including the time required for the publishing area to serve the request if needed. Then, $E[D_c]$ is given by:

$$E[D_c] = \left( \sum_{i=1}^{M} E[D_{c,i}(T_{c,i})] \prod_{j=i+1}^{M} R_{c,j}(T_{c,j}) \right) + C(\hat{\Lambda}_c) \prod_{j=1}^{M} R_{c,j}(T_{c,j}), \tag{19}$$

where $C(\hat{\Lambda}_c)$ is the mean cost (measured in time units) to retrieve a content at the publishing area as a function of the load $\hat{\Lambda}_c$. Recall that tier 1 (resp., tier $M$) is the closest to the custodians (resp., users). Therefore, $\prod_{j=i+1}^{M} R_{c,j}(T_{c,j})$ corresponds to the fraction of requests to content $c$ that reach tier $i$, for $i = 1, \ldots, M - 1$.

5. Parameter Tuning

In this section we consider the problem of minimizing average delay under average storage constraints. To this aim, we use the stateful model that was introduced in the previous section. While in Section 4 the analysis targeted a single tagged content, in this section we account for the limited space available in the caches and for contents that compete for cache space.
To simplify presentation, we consider a single tier \( M = 1 \). We also assume that the delays experienced by requests at the custodian are given and fixed, equal to \( C \).

Let \( D_c \) denote the delay experienced by a requester of content \( c \). \( E[D_c] \) is obtained by substituting (14) into (19),

\[
E[D_c] = (1 - \pi_c) \left( \frac{1 - e^{-\gamma \pi_c T_c}}{\pi_c \gamma} + Ce^{-\gamma \pi_c T_c} \right)
\]

and

\[
E[D] = \sum_{c=1}^{C} \frac{\lambda_c}{\lambda} E[D_c]
\]

Let \( \alpha_c = 1/\mu_c \), \( \alpha = (\alpha_1, \alpha_2, \ldots, \alpha_C) \) and \( T = (T_1, T_2, \ldots, T_C) \). In light of (1) and (20)-(21), we pose the following joint placement and search optimization problem:

\[
\min_{(\alpha,T)} \quad E[D] = \sum_{c=1}^{C} \frac{\lambda_c}{\lambda} (1 - \lambda_c \alpha_c) \left( \frac{1 - e^{-\gamma \lambda_c \alpha_c T_c}}{\lambda_c \alpha_c \gamma} + Ce^{-\gamma \lambda_c \alpha_c T_c} \right)
\]

\[
s.t. \quad \sum_{c=1}^{C} \lambda_c \alpha_c = B
\]

Note that we impose a constraint on the expected buffer size, i.e., the number of expected items in the cache cannot exceed the buffer size \( B \). Similar constraint has been considered, for instance, in [39]. Moreover, recent work [34] shows that, for TTL caches, we can size the buffer as \( B(1 + \epsilon) \), where \( B \) (resp., \( \epsilon \)) grows in a sublinear manner (resp., shrinks to zero) with respect to \( C \), and content will not need to be evicted from the cache before their timers expire, with high probability.

The reinforced counter vector \( \alpha \) impacts content placement, while the random walk vector \( T \) impacts content search. By jointly optimizing for placement and search parameters, under storage constraints, we obtain insights about the interplay between these two fundamental mechanisms.

In what follows, we do not solve the joint optimization problem directly. Instead, to simplify the solution, we solve two problems independently: first, we consider
the optimal placement given a search strategy, and then the optimal search given a pre-determined placement. In our case studies we discuss the impact of these simplifications.

5.1. Optimal Placement Given Search Strategy

We first address the optimal placement problem, that is we determine how the buffer space at the cache-routers should be statistically divided among the contents to optimize the overall performance.

5.1.1. Special Case: \( T \) large

We begin by considering large time to live values. In the limit when \( T_c = \infty \), the time spent locally searching for a content is unbounded. Under this assumption, the optimization problem stated in (22) reduces to

\[
\min_{\pi} \sum_{c=1}^{C} \frac{\lambda_c}{\lambda} (1 - \pi_c) \left( \frac{1}{\pi_c \gamma} \right) \\
\text{s.t.} \sum_{c=1}^{C} \pi_c = B
\]

We construct the Lagrange function,

\[
\mathcal{L}(\pi, \beta) = \sum_{c=1}^{C} \frac{\lambda_c}{\lambda} \left( 1 - \pi_c \right) \left( \frac{1}{\pi_c \gamma} \right) + \beta \left( \sum_{c=1}^{C} \pi_c - B \right)
\]  

(24)

where \( \beta \) is a Lagrange multiplier. Setting the derivative of the Lagrangian with respect to \( \pi_c \) equal to zero and using (23) yields,

\[
\beta = \frac{\left( \sum_{c=1}^{C} \sqrt{\lambda_c} \right)^2}{\gamma \lambda B^2}.
\]

(25)

Therefore,

\[
\pi_c = B \frac{\sqrt{\lambda_c}}{\left( \sum_{c=1}^{C} \sqrt{\lambda_c} \right)}, \quad c = 1, \ldots, C.
\]

(26)
When $B = 1$, the optimal policy (26) is the square-root allocation proposed by Cohen and Shenker [19] in the context of peer-to-peer systems. It is interesting that we obtain a similar result for the cache network under study. This is because in both cases the optimization problem can be reformulated as to minimize $\sum_{c=1}^{C}(\lambda_c/\lambda)/\pi_c$ under the constraint that $\sum_{c=1}^{C} \pi_c = B$. In [19] the term $1/\pi_c$ is the mean time to find content $c$, which is the average of a geometric random variable with probability of success $\pi_c$. In the cache network under study, the term $1/\pi_c$ follows from expression (20).

5.1.2. Special Case: $T = 0$

Next, we consider the case $T = 0$. When a request for $c$ arrives at a cache-router and does not find the content, the request is automatically sent to the next level in the hierarchy of tiers. Then, the optimization problem reduces to

$$\min \pi \quad E[D] = \sum_{c=1}^{C} \frac{\lambda_c}{\lambda} (1 - \pi_c) C$$

s.t. $\sum_{c=1}^{C} \pi_c = B$ \hspace{1cm} (27)

In this case, the optimal solution consists of ordering contents based on $\lambda_c$ and storing the $B$ most popular ones in the cache, i.e., $\pi_c = 1$ for $c = 1, \ldots, B$ and $\pi_c = 0$ otherwise. Note that this rule was shown to be optimal by Liu, Nain, Niclausse and Towsley [40] in the context of Web servers.
5.1.3. Special Case: $\gamma T$ small

For $\gamma T \ll 1$, we have $e^{-\gamma \pi c T} \approx 1 - \gamma \pi c T$. The optimization problem is given by

$$\min_{\pi} \quad E[D] = \sum_{c=1}^{C} \frac{\lambda_c}{\lambda} (1 - \pi_c) (T + (1 - \gamma \pi c T) C)$$

$$\text{s.t.} \quad \sum_{c=1}^{C} \pi_c = B$$
$$0 \leq \pi_c \leq 1$$

Note that the objective function can be rewritten as

$$E[D] = \sum_{c=1}^{C} \frac{\lambda_c}{\lambda} \left( CT \gamma \pi c^2 + (-C + T) - CT \gamma \right) \pi_c + (C + T)$$

This is a special separable convex quadratic program, known as the economic dispatch problem [41] or continuous quadratic knapsack [42]. It can be solved in linear time using techniques presented in [43]. Alternatively, in Appendix B we present the dual of the problem above, which naturally yields a simple interactive gradient descent solution algorithm.

5.2. Optimal Search Given Placement

In this section we address the optimal search problem, that is the choice of the $T_c$’s, when placement is given (the $\pi_c$’s have been determined). Then, the problem reduces to

$$\min_{T} \quad \sum_{c=1}^{C} \frac{\lambda_c}{\lambda} (1 - \pi_c) \left( \frac{1 - e^{-\gamma \pi c T_c}}{\gamma \pi c} + Ce^{-\gamma \pi c T_c} \right)$$

$$\text{s.t.} \quad T_c \geq 0, \ c = 1, \ldots, C$$

For each content $c$ the function to be minimized is $f(T)$,

$$f(T) = \frac{1}{\gamma \pi c} \left( 1 - e^{-\gamma \pi c T} \right) + Ce^{-\pi c \gamma T}$$
and
\[
\frac{df(T)}{dT} = e^{-\gamma \pi_c T} - \gamma \pi_c \mathcal{C} e^{-\gamma \pi_c T} \tag{36}
\]

For a given content \(c\), a random walk search should be issued with \(T = \infty\) whenever \(df(T)/dT < 0\), i.e., if \(1 - \gamma \pi_c \mathcal{C} < 0\). Otherwise, the request for content \(c\) should be sent directly to the publishing area;

\[
T_c = \begin{cases} 
\infty, & \pi_c > 1/(\mathcal{C} \gamma) \\
0, & \text{otherwise}
\end{cases} \tag{37}
\]

**Remarks:** Although we do not solve the joint placement and search optimization problem, the special cases considered above provide some guidance for system tuning. The studies we conduct in the following section provide evidence of the usefulness of our model solutions. In addition, we may try different approximation approaches to solve the combined placement and search problem. For instance, one such approach is to first optimize for the \(\pi_c\)'s assuming \(T\) is large and set \(T_c = \infty\) for all contents that satisfy \(\pi_c > 1/(\mathcal{C} \gamma)\) (see (37)). Then, set \(T_c = 0\) for the contents for which \(\pi_c \leq 1/(\mathcal{C} \gamma)\), and recompute \(\pi_c\) for such contents using the solution presented in Section 5.1.2 so as to fill the available buffer space. The performance of this and other heuristics is subject for future research.

6. Evaluation

In this section we report numerical results obtained using the proposed model. Our goals are a) to show tradeoffs involved in the choice of the *time to live* (TTL) parameter, b) to illustrate the interplay between content search and placement, and c) to numerically solve the optimization problems posed in this paper, giving insights about the solutions. In Sections 6.1 and 6.2 we consider the stateless model, and in Section 6.3 the stateful model is examined. The topologies considered are illustrated
in Figure 2, and comprise multiple domains, wherein all cache-routers inside a domain are virtually fully connected.

6.1. Tradeoff in The Choice of TTL: Single Content Scenario

Our goal is to illustrate the trade-off between hit rate and expected delay (additional examples illustrating related tradeoffs were reported in [44]). Let $\Lambda_c = 1$. We consider a single content that is to be served in the three-tiered topology shown in Figure 2(b). We use the same assumptions as those introduced by the stateless model (Section 4.2.1) with $N$ large. In addition, we assume that $\pi$ and $T$ are equal at the three considered domains (this assumption will be removed in the other scenarios studied).

As requests are filtered towards the custodian, the rate of requests decreases when moving from tier 3 to tier 1. The rate at which reinforced counters are decremented also decreases from tier 3 to tier 1, in order to keep $\pi$ constant.

![Illustrative topology](image)

Figure 2: Illustrative topology
Figure 3(a) shows the the expected delay to reach the custodian and the custodian load for different values of $\pi$ and $T$. For a given value of $\pi$, the dotted lines indicate that as $T$ increases the load at the custodian decreases and the expected delays in the domains increases. In contrast, for a given value of $T$, as $\pi$ increases, content becomes more available, which causes a decrease in the load at the custodian and in the expected delay.

Next, our goal is to evaluate the expected delay. To this aim, we use an M/M/1 queue to model the delay at the custodian. We let the custodian cost be given by $C(\hat{\Lambda}_c) = 1/(0.9 - \hat{\Lambda}_c)$, which corresponds to the delay of an M/M/1 queue with service capacity of 0.9.

Figure 3(b) shows how the expected delay (obtained with equation (19)) varies as a function of $\pi$ and $T$. For $\pi = 0.05$ and $\pi = 0.1$, as $T$ increases, the expected delay $E[D_c]$ first decreases and then increases. The initial decrease occurs due to a decrease in the custodian load. Nonetheless, as $T$ further increases the gains due to decreased load at the custodian are dominated by the increased expected delay before reaching the custodian. The optimal value of $T$ is approximately 1.5 and 0.5 for $\pi$ equal to 0.05 and 0.1, respectively.

6.2. Benefits of Load Aggregation

While in the previous section we studied the dynamics of a single content, now we consider four content popularities: very low, low, medium and high. In Figures 4 and 5 we plot expected delay both for the one-tiered architecture (Figure 2(a)) and the three-tiered architecture (Figure 2(b)). The one-tiered architecture has 3850 cache-routers. In the three-tiered architecture we assume that each domain at tiers 1, 2 and 3 contains 350 cache-routers. Tier 1 has eight domains (2800 cache-routers), tier 2 has 2 domains (700 cache-routers) and tier 3 has one domain (350 cache-
Figure 3: Scatter plot indicating the tradeoff in the choice of TTL \( T \): (a) larger values of \( T \) reduce load in custodian at cost of increased expected delay in domain; (b) expected delay as a function of expected delay in domains, assuming cost at custodian \( C(\hat{\Lambda}_c) = 1/(0.9 - \hat{\Lambda}_c) \).
routers). We define the same number of routers in both architectures so that the comparison is fair.

The request arrival rate for each type of content was obtained from real data collected from a major Brazilian broadband service provider [35]. The content request rates are $\lambda_1 = 0.8$, $\lambda_2 = 0.5$, $\lambda_3 = 0.1$ and $\lambda_4 = 0.01$ req/sec. The Request Counter (RC) of each content is decremented at constant rate $\mu = 1$ in the three tiers. The value of $\pi$ varies for each content in each tier due to the fact that content requests are filtered out as they travel towards the custodian. In addition, we assume that $T$ is equal in the three domains and the mean time to retrieve a content from the publishing area exponentially increases with respect to the amount of requests hitting the publishing area, $\mathcal{C}(\hat{\Lambda}_c) = e^{\hat{\Lambda}_c}$.

Figures 4 and 5 show the benefits of load aggregation that occurs in the three-tiered architecture: requests that are not satisfied in tier three are aggregated in the second and third tiers. Aggregation increases the probability to find the content in these tiers. We observe that contents with low and medium popularities benefit the most from load aggregation. Note that the expected delay decreases by several orders of magnitude for low popularity contents when we consider a three-tiered architecture. For very low and high popularity contents, a significant reduction is not observed. For highly popular contents, the probability to store the content in at least one of the tiers is high in both architectures, and only a small fraction of the requests is served by the publishing area. For very low popularity contents, the opposite occurs: the majority of requests are served by the publishing area, as the probability that content is stored in one of the tiers is very low.

Figures 4 and 5 show that the three-tiered architecture yields lower delays, for all content popularities. Next, we consider the optimal TTL choice in the three-tiered topology. For very low popularity contents, the best choice is $T = 0$ as the majority
Figure 4: Expected Delay: very low and low popularity contents

Figure 5: Expected Delay: medium and high popularity contents
of requests must be served by the publishing area. For high popularity contents, the best choice is also $T = 0$ because the probability to find the content in the first router of the domain is very high. On the other hand, for low popularity contents, Figure 4 shows that the mean delay is minimized when $T \approx 0.1$.

6.3. Validation of the Optimal Solution

Next, we consider a single domain under the assumption of the stateful model (Section 4.2.2, with large $N$). Our goal is to obtain the values of $\pi_c$ and $T_c$, $c = 1, 2, 3$, that minimize expected delay. We consider three contents with high, medium and low popularity sharing a memory that can store, on average, one replica of content, $B = 1$. The publisher cost is $C = 10$, the random search time is $1/\gamma = 40$ ms and the content request rates are $\lambda_1 = 0.8$, $\lambda_2 = 0.1$ and $\lambda_3 = 0.002$ req/sec. As in the previous section, content popularities were inspired by data collected from a major Brazilian broadband service provider [35].

Using (21), we compute the expected delay for different values of $\pi_c$ and $T_c$, $\pi_c$ varying from 0.01 to 0.99 and $T_c$ varying from 0 to 30s, $i = 1, 2, 3$. The results of our exhaustive search for the minimum delay are reported in Figure 6. Figure 6(a) shows the minimum average delay attained as a function of $\pi_1$, $\pi_2$ and $\pi_3$, considering all possible values of the other parameters. Similarly, Figure 6(b) shows the minimum attainable average delay as a function of $T_1$, $T_2$ and $T_3$.

In Section 5.1, when considering the case $T = 0$ we showed that (26) yields the optimal values of $\pi_c$. For our experimental parameters, (26) yields $\pi_1 = 0.71$, $\pi_2 = 0.25$ and $\pi_3 = 0.04$. These values are very close to the three points that minimize the expected delay obtained using the exhaustive search, as shown in Figure 6(a), which indicates the usefulness of the closed-form expressions derived in this paper. Even though the solutions we obtained do not account for joint search and placement.
(a) Minimum expected delay is obtained for $\pi_1 = 0.71$, $\pi_2 = 0.25$ and $\pi_3 = 0.04$

(b) Minimum expected delay is obtained for $T_1 \geq 0.2$, $T_2 \geq 0.2$ and $T_3 \geq 0.2$

Figure 6: Minimum expected delay for each value of $\pi_c$ and $T_c$.

they yield relevant guidelines that can be effectively computed in a scalable fashion. The exhaustive search for solutions took us a few hours using a Pentium IV machine, whereas the evaluation of the proposed closed-form expressions takes a fraction of
7. Discussion

7.1. Joint Placement and Search Optimization

In this paper, we introduced a new architecture, followed by a model and its analysis that couples search through random walks with placement through reinforced counters to yield simple expressions for metrics of interest. The model allows us to pose an optimization problem that is amenable to numerical solution. Previous works considered heuristics to solve the joint placement and search problem [45, 46, 47], accounting for the tradeoff between exploration and exploitation of paths towards content replicas [24]. To the best of our knowledge, we are the first to account for such a tradeoff using random walks, which have previously been proposed in the context of peer-to-peer systems as an efficient way to search for content [10]. We are also not aware of previous works that generalize the cache utility framework [48, 49] from a single cache to a cache network setting.

7.2. Threats to Validity

In this section we discuss some of the limitations and simplifying assumptions, as well as extensions subject for future work.

7.2.1. Threats to Internal Validity

The parameters used in numerical evaluations serve to illustrate different properties of the proposed model. It remains for one to apply the proposed framework in a realistic setting, showing how to make it scale for hundreds of contents whose popularities vary over time. Section 5 provides a first step towards that goal.
7.2.2. Threats to External Validity

In this paper, we consider a simple setup which allows us to obtain an analytical model amenable to analysis. The extension to caches with TTL replacement policy, as well as other policies such as LRU, FIFO and Random, is a subject for future research.

We have focused on the placement and search strategies. We assumed throughout this paper the ZDD assumption (zero delay for downloads). Accounting for the effects of service capacities for download on system performance is out of the scope of this work.

8. Conclusion

Content search and placement are two of the most fundamental mechanisms that must be addressed by any content distribution network. In this paper, we have introduced a simple analytical model that couples search through random walks and placement through a TTL-like mechanism. Although the proposed model is simple, it captures the key tradeoffs involved in the choice of parameters. Using the model, we posed an optimization problem which consists of minimizing the expected delay experienced by users subject to expected storage constraints. The solution to the optimization problem indicates for how long should one wait before resorting to custodians in order to download the desired content. We believe that this paper is a first step towards a more foundational understanding of the relationship between search and placement, which is key for the efficient deployment of content centric networks.
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Appendix A. Cache Insertion Rate

In this appendix we study the rate at which content is inserted into cache. Recall that associated with each counter and content there is a threshold $K$, such that when the reinforced counter exceeds $K$, the corresponding content must be stored into cache. Next, we consider the impact of $K$ on the cache insertion rate. The cache insertion rate for a given content is the rate at which that content is brought into the cache. Similarly, the cache eviction rate is the rate at which content is evicted from the cache. Due to flow balance, in steady state the cache insertion rate equals the cache eviction rate.

Let $\psi_c$ be the insertion rate. Recall that $\lambda_c$ and $\mu_c$ are the request arrival rate for content $c$ and the rate at which the counter associated to content $c$ is decremented, respectively (Table 1). Then

\[
\psi_c = \lambda_c \rho_c^K (1 - \rho_c) = \pi_c (\mu_c - \lambda_c)
\] (A.1)

Recall that the content miss rate is given by $\lambda_c \sum_{i=0}^{K} \rho_c^i (1 - \rho_c) = \lambda_c (1 - \pi_c)$. We note that, except for $K = 0$, the content insertion rate is strictly smaller than the content miss rate.

Let us now consider the impact of $K$ on the insertion rate, assuming a constant miss rate. For a given miss rate, $\pi_c$ is determined. Once $\pi_c$ is established, it follows from (1) that larger values of $K$ yield smaller values of $\mu_c$. A decrease in $\mu_c$, in turn, causes a reduction in the insertion rate (see eq. (A.1)).
A smaller insertion rate, for the same hit ratio, has several advantages: (a) first, increasing the number of cache writes slows down servicing the requests for other contents, that is, cache churn increases which reduces throughput \([50, 4, 51, 16]\); (b) if flash memory is used for the cache, write operations are much slower than reads; (c) writes wear-out the flash memory; and (d) additional writes mean increasing power consumption.

Reducing the cache eviction rate might also lead to a reduction in network load. To appreciate this point, consider a scenario similar to the one presented in \([52]\). A custodian is connected to a cache through one route, and to clients through another separate route. The link between the custodian and the cache is used only when a cache insertion is required. The link between the custodian and the clients, in contrast, is used after every cache miss, irrespectively of whether the cache miss resulted in a cache insertion. In this case, reducing the cache insertion rate produces a reduction in the load of the link between the custodian and the cache.

In summary, larger values of \(K\) favor a reduction in the insertion rate, which benefits system performance. The impact of \(K\) is similar in spirit to that of \(k\) in \(k\)-LRU \([16]\) and \(N\) in \(N\)-hit caching \([51]\).

**Appendix B. Dual Problem For \(\gamma T \ll 1\)**

Let

\[
K_{2,i} = \frac{\lambda_i}{\lambda} CT\gamma
\]

(B.1)

\[
K_{1,i} = \frac{\lambda_i}{\lambda} \left( -(C + T) - CT\gamma \right)
\]

(B.2)

\[
K_{0,i} = (C + T)
\]

(B.3)
Let \(1\) be a row vector of ones. The optimization problem posed in Section 5.1.3 can be stated as a quadratic program,

\[
\begin{align*}
\min & & \frac{1}{2} \pi^T Q \pi + c^T \pi \\
\text{s.t.} & & A \pi \leq b \\
& & 1 \pi = B
\end{align*}
\]  

(B.4)

(B.5)

(B.6)

where \(Q\) is a diagonal matrix with \(Q(i,i) = 2K_2i\), \(c\) is a vector with \(c(i) = K_1i\) and

\[
A = \begin{bmatrix}
1 & 0 & \cdots & 0 \\
0 & \ddots & \cdots & \vdots \\
\vdots & \ddots & \ddots & \ddots \\
0 & \cdots & 0 & 1 \\
-1 & 0 & \cdots & 0 \\
0 & \ddots & \ddots & \vdots \\
\vdots & \ddots & \ddots & 0 \\
0 & \cdots & 0 & -1
\end{bmatrix}, \quad b = \begin{bmatrix}1 \\
1 \\
\vdots \\
1 \\
1 \\
0 \\
0 \\
\vdots \\
0
\end{bmatrix}
\]

(B.7)

Note that because \(Q\) is a positive-definite matrix, there is a unique global minimizer [53].

Let \(\delta = (\nu, \upsilon)\), where \(\nu_i\) and \(\upsilon_i\) are the Lagrange multipliers associated with the constraints \(\pi_i \leq 1\) and the non-negativity constraint \(\pi_i \geq 0\), \(i = 1, \ldots, C\), respectively. The Lagrangian is given by

\[
\mathcal{L}(\pi, \delta, \epsilon) = \frac{1}{2} \pi^T Q \pi + c^T \pi + \delta^T (A \pi - b) + \epsilon (1 \pi - B)
\]  

(B.8)

\[
= \frac{1}{2} \sum_{i=1}^{C} \pi_i^2 q_i + \sum_{i=1}^{C} c_i \pi_i + \sum_{i=1}^{C} \nu_i (\pi_i - 1) + \sum_{i=1}^{C} \nu_i (-\pi_i) + \epsilon \left( \sum_{i=1}^{C} \pi_i - B \right)
\]  

(B.9)

\[
= \sum_{i=1}^{C} \pi_i \left( \frac{q_i \pi_i}{2} + c_i + \nu_i - \nu_i + \epsilon \right) - \left( \sum_{i=1}^{C} \nu_i \right) \epsilon - B
\]  

(B.10)
To determine the dual function $g(\delta, \epsilon)$, defined as

$$g(\delta, \epsilon) = \inf_{\pi} \mathcal{L}(\pi, \delta, \epsilon)$$  \hspace{1cm} (B.11)

we note that

$$\nabla_{\pi} \mathcal{L}(\pi, \delta, \epsilon) = 0 \Rightarrow \pi^* = -Q^{-1}(A^T\delta + c + 1^T\epsilon)$$  \hspace{1cm} (B.12)

Then,

$$\pi_i^* = - \frac{1}{q_i} (c_i + \nu_i - v_i + \epsilon)$$  \hspace{1cm} (B.13)

The dual function is

$$g(\delta, \epsilon) = -\frac{1}{2} \sum_{i=1}^{C} (\pi_i^*)^2 q_i - \left( \sum_{i=1}^{C} \nu_i \right) - \epsilon B$$  \hspace{1cm} (B.14)

The dual problem is also a quadratic program,

$$\max_{\epsilon, \delta} \quad -\frac{1}{2} (\pi^*)^T Q \pi^* - b^T \delta - B \epsilon$$ \hspace{1cm} (B.15)

$$s.t. \quad \delta \geq 0$$ \hspace{1cm} (B.16)

The dual problem naturally yields an asynchronous distributed solution [54].

**Appendix C. Stateful Model**

Two possible ways to implement stateful searches are: (a) when an inter-domain request arrives at a router and finds that the request cannot be immediately satisfied, a search is initiated and the searcher pre-selects $j$ out of the remaining $N - 1$ routers to conduct the search or; (b) after the search is initiated, the searcher chooses the next router to visit uniformly at random, from those that have not yet been visited before.
In this Appendix we consider the case in which routers are pre-selected at the beginning of the search. We assume that $\gamma$ is very large compared to the rate at which RCs are updated.

Let $J$ be a random variable denoting the number of routers to be visited by time $t$ excluding the first visited router, and as before, let $L_c$ be the number of replicas of content $c$ in the domain under consideration. Note that as we do not allow revisits, $J \leq N-1$. Conditioning on $J=j$ visited routers and $L_c = l$ content replicas present in the $N-1$ possible caches to visit,

$$\tilde{R}_c(t|J = j, L_c = l) = (1 - \pi_c) \frac{(N-1-l)}{(N-1)}. \quad \text{(C.1)}$$

We assume, like in Section 4.2.1, that the search is sufficiently fast compared to the rate at which content is replaced. Replacing (C.1) into (3),

$$\tilde{R}_c(t|J = j) = \sum_{l=0}^{N-1} \tilde{R}_c(t|J = j, L_c = l) \binom{N-1}{l} \pi_c^l (1 - \pi_c)^{N-1-l} \quad \text{(C.2)}$$

$$= \sum_{l=0}^{N-1-j} \tilde{R}_c(t|J = j, L_c = l) \binom{N-1}{l} \pi_c^l (1 - \pi_c)^{N-1-l} \quad \text{(C.3)}$$

$$= (1 - \pi_c) \sum_{l=0}^{N-1-j} \binom{N-1-j}{l} \pi_c^l (1 - \pi_c)^{N-1-l} \quad \text{(C.4)}$$

$$= (1 - \pi_c)^{j+1}. \quad \text{(C.5)}$$

(C.3) follows from (C.2) since $\tilde{R}_c(t|J = j) = 0$ if $l > N-1-j$ as at least one of the $j$ routers necessarily has the content.

It is interesting to observe that (C.5) and (9) are identical, although derived from two different sets of assumptions.
Appendix D. Dependencies Among Caches, Miss Stream Behavior and Poisson Assumption

Next, we study properties of the miss stream of a single RC cache (Appendix D.1), and dependencies that arise when connecting multiple caches (Appendix D.2). We consider the case $K = 0$.

Appendix D.1. Inter-request Times in Miss Stream: Single Cache

Next, we study the inter-request times in the miss stream for a given tagged content with arrival rate $\lambda$, accounting for a cache governed by a reinforced counter whose counter is decremented at rate $\mu$. We adopt the same notation as in Sections 2 and 3 of [33]. Let $B$ (resp., $B(s)$) be the busy period (resp., the Laplace-Stieltjes transform of the busy period) of an M/M/1 queue. $B(s)$ is given by (see for instance [55]),

$$B(s) = \frac{1}{2\lambda} \left( \lambda + \mu + s - \sqrt{(\lambda + \mu + s)^2 - 4\lambda\mu} \right)$$  \hspace{1cm} (D.1)

The inter-miss times form a renewal process. Let the random variable characterizing the inter-miss times be denoted by $Y$ and let $G^*(s)$ and $G(t)$ be its Laplace transform and CDF, respectively. Let $m_r$ be the stationary miss rate. Then, $m_r = 1/E[Y] = \lambda \pi_0$.

The inter-miss times of an RC cache are related to the busy period of an M/M/1 queue as follows,

$$Y = B + X$$  \hspace{1cm} (D.2)

where $X$ is an exponentially distributed random variable with rate $\lambda$. Note that

$$G^*(s) = B^*(s)X^*(s) \hspace{1cm} (D.3)$$

$$= \frac{1}{2\lambda} \left( \lambda + \mu + s - \sqrt{(\lambda + \mu + s)^2 - 4\lambda\mu} \right) \frac{\lambda}{\lambda + s} \hspace{1cm} (D.4)$$

44
Then,

\[
\frac{1}{E[Y]} = m_r = \left( -\frac{dG^*(0)}{ds} \right)^{-1} = \lambda \left( 1 - \frac{\lambda}{\mu} \right)
\]

The request rate for the tagged content at the miss stream is \(\lambda \pi_0\) (see (D.7), where \(\pi_0 = (1 - \lambda/\mu))\).

It follows from (D.4) that the miss stream is not characterized by a Poisson process, which motivates the discussion presented in Appendix D.2.

**Appendix D.2. Inter-domain Requests: Numerically Assessing the Impact of the Poisson Assumption**

Next, our goal is to assess the impact of approximating the inter-domain request processes at different caches for each content as independent Poisson processes. Previous works considered, in a different context, the empirical assessment of the impact of the independence assumption, sometimes referred to as *Kleinrock’s Independence assumption* [56].

For illustrative purposes, we consider a simple setup shown in Figure D.7. We then simulate users requests for a given content arriving at the bottom tier according to a Poisson process, and investigate the arrival processes at higher tiers. The numerical results reported in this section are obtained using the Tangram II tool [57].

As shown in Figure D.7, the network of cache-routers is tripartite and has three tiers and a publishing area. This topology resembles, for instance, the one used by Akamai [58]. Each tier has one domain with four cache routers. Users requests at the first tier arrive at each cache router according to a Poisson process with rate \(\lambda\). We
vary the rate $\lambda$ to consider high ($\lambda = 0.9$), medium ($\lambda = 0.25$) and low ($\lambda = 0.0025$) popular contents. We consider the reinforced counter decrement rate $\mu$ equal to 1 and the reinforced counter threshold $K$ equal to 0. We let TTL equal to zero, i.e., if the content is not found in the first visited cache router of a domain, the request is immediately forwarded to the next tier. The cache router selected at the next tier is chosen uniformly at random.

We compute two measures in the simulation: the cache hit probability as perceived by the arrivals, i.e., the fraction of requests that find the content at cache and the miss stream request rate at each cache router, i.e., the rate of requests that do not find the content in the cache. We compared these measures with those computed with the analytical model under the assumption of Poisson arrivals.

The confidence level considered in all simulations is 95%. In the majority of the results, the measures calculated with the analytical model are within the confidence interval computed by the simulation.

Figures D.8, D.9 and D.10 show the cache hit probability and the miss stream
request rate of each cache router obtained for high, medium and low popular contents. The results show that the analytical model is more accurate for high and low popular contents, and that the length of the confidence intervals varies between 3% and 13% of the sample mean.

It is well known that the superposition of a large number of independent flows produces a Poisson flow. Randomness and thinning also make the traffic look like Poisson [59]. We believe that these are some of the factors that may make the inter-domain traffic look like Poisson. A broader and more thorough assessment of the scenarios for which the Poisson assumption is applicable is subject for future work.

(a) Cache Hit Probability

(b) Miss Stream Request Rate

Figure D.8: High Popular Content.
(a) Cache Hit Probability
(b) Miss Stream Request Rate

Figure D.9: Medium Popular Content.

(a) Cache Hit Probability
(b) Miss Stream Request Rate

Figure D.10: Low Popular Content.
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