Multi-path multi-flow entanglement routing in a quantum network

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Introduction—Quantum repeaters at nodes of the future computer network will permit distant parties to generate high-rate shared entanglement. Shared entanglement is the fundamental enabler of a wide swath of distributed quantum computation and physics-based information-security protocols, including quantum key distribution (QKD). With multiple source-destination pairs and groups attempting to generate shared entanglement over a common underlying repeater network, a multilayer networking protocol stack will need to be developed for the (quantum memory and processing enabled) repeater nodes to perform local quantum operations and classical data exchange with other nodes in order to schedule and route multiple flows while maximizing entanglement throughput and minimizing latency. Further, given a total resource cost constraint (e.g., number, size, coherence times of quantum memories), the optimal network design, i.e., where to place the repeaters and which ones to connect to maximize rate performance is a wide open question. In this paper, for a network of repeaters connected via lossy optical links, we consider the problem of developing quantum routing protocols based on quantum memories and probabilistic Bell measurements, to maximize the simultaneously-attainable rate region for multiple flows of pairwise entanglement generation. We show that even using local link-state knowledge at nodes, a pair of users can achieve a large multi-path routing gain, i.e., over the rate attainable with a linear repeater chain along the shortest path. Additionally, we present conceptually simple multi-flow repeater protocols that outperform the rate region attainable by the rate attainable with a classical max-flow min-cut problem with link capacities specified on the edges.

Prior work—Consider the aforesaid problem with just the channel transmissivities specified on the edges \( \eta_1, \ldots, \eta_M \) but with no further constraints imposed on the nodes, other than their being limited by general fully-error-corrected quantum computers. Pirandola et al. showed that for a single flow \( K = 1 \), the maximum entanglement-generation rate \( R_1 \) reduces to the classical max-flow min-cut problem with link \( i \) being associated with capacity \( C_i = -\log_2(1 - \eta_i) \) ebits per channel use, \( 1 \leq i \leq M \), where \( C_i \) is the maximum rate at which shared entanglement can be generated with any point-to-point protocol over a link of transmissivity \( \eta_i \) with unlimited two-way authenticated classical communication. Pirandola subsequently argued that clas-
FIG. 1. Schematic of a continental-scale quantum repeater network. The large (green) circles represent ‘trusted’ nodes, which are connected via a classical network. The blue circles denote repeater stations, and the red circles inside them represent quantum memories. The thin lines connecting the red circles are lossy optical (fiber) channels. In principle, all nodes in the network could be equipped with quantum repeaters (i.e., no trusted nodes), in which case depending upon the need, a node can be a consumer of shared entanglement, or act a router to conduit entanglement generation flows between other nodes.

FIG. 2. Quantum network with a square grid topology. Every cycle of the protocol consists of two parts: (a) in the first part, edges between neighboring repeaters are attempted which succeed with probability $p$ (dashed lines), and (b) in the second, internal edges (entanglement swaps) are attempted (with the objective of creating an unbroken line between Alice and Bob) which succeed with probability $q$.

Our main contributions are summarized below:

1. Multipath routing gain—A major challenge in building quantum repeaters is that photon losses (on the lossy optical channels) connecting repeater stations and the sub-unity probability of entanglement swapping at each repeater node (due to imperfect device efficiencies combined with the intrinsic probabilistic nature of BSMs implemented via linear optics) leads to an exponential drop-off in the entanglement rate with distance [6, 7]. It is however known that a linear repeater chain connecting the communicating parties Alice and Bob can attain a better exponent in the (exponential) rate-distance scaling compared the best conceivable repeater-less (direct-transmission) protocols, even when the repeaters are built with lossy devices [6, 7]. We find that the presence of multiple paths between Alice and Bob in a repeater network with a more connected topology (e.g., the square-grid topology that we study) enables an even-further improved scaling of the rate vs. distance as compared with a linear chain of repeaters laid out along a shortest path connecting Alice and Bob. Interestingly, a substantial multi-path routing advantage can be had even when every repeater station in the network only has local link-state knowledge (i.e., whether the neighboring links were successful in establishing an entangled pair across them in a given time slot), and with losses on the edges and nodes accounted for. Our analysis is done for a square-grid network topology shown in Fig. 2, and the rate-distance tradeoffs are summarized in Fig. 3.

2. Multiflow entanglement routing—We study simultaneous entanglement generation between multiple Alice-Bob pairs, and present a protocol that significantly outperforms the ‘rate region’ attainable compared with simply time-sharing the aforesaid single-flow multi-path protocol over the multiple flows (Fig. 5). This protocol also only employs local link-state knowledge.
FIG. 3. Entanglement generation rate as a function of the Alice-Bob separation along X and Y (on a square grid) as a function of (p, q); (a) \( R^{(UB)}(0.6) \) is the distance-independent Pirandola rate upper bound for \( p = 0.6 \), achieving which requires perfect quantum processing power at nodes. \( R_s(p, q) \) is the rate attained by a global-knowledge protocol we propose where each node, in each time step, knows whether any node in the entire network succeeded or failed to establish entanglement. \( R_s(0.6, 1) \) is also distance independent, and within a factor 3.6 of \( R^{(UB)}(0.6) \). With \( q < 1 \), e.g., \( R_s(0.6, 0.9) \), the rate decays exponentially with distance. \( R^{(UB)} \) is an upper bound on the rate attainable with global-knowledge. (c) \( R_{loc} \) is attained by a protocol we propose where each node, in each time step, only needs to know the state of neighboring edges. The rate-distance scaling exponent of \( R_{loc} \) is significantly superior to that of a linear repeater chain along the shortest path, \( R_{lin} \), demonstrating the multipath routing advantage.

3. Exact multiflow rate region for tree topology—For the case when the graph topology \( G \) is a tree, and with any number \( (K > 1) \) of simultaneous flows, we are able to exactly evaluate the optimum rate region, as well as specify a protocol that achieves this optimum rate (using global link-state knowledge).